Math 10A with Professor Stankova
Worksheet, Discussion #16; Monday, 10/2/2017
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Anti-Derivatives

Example

1. Find an antiderivative of $\dfrac{1}{2x}$.

> **Solution:** By the constant integration law, we know that an antiderivative of $\dfrac{1}{2x}$ is half an antiderivative of $\dfrac{1}{x}$, thus it suffices to find an antiderivative of $\dfrac{1}{x} = x^{-1}$. Ideally, we would take $\dfrac{x^{n+1}}{n+1}$ for an antiderivative of x^n, but $-1+1=0$ so we cannot use the power antiderivative rule. But, we remember that the derivative of $\ln x$ is $\frac{1}{x}$. So, an antiderivative of $\dfrac{1}{2x}$ is $\dfrac{\ln x}{2}$.

2. True **FALSE** Just like differentiation where we can use the chain rule/product rule/quotient rule/etc. to always be able to find the derivative of a function, we can find similar rules to do the same with finding an anti-derivative.

3. True **FALSE** There exists a unique anti-derivative.

Problems

4. Find an antiderivative of $5e^x$.

> **Solution:** An antiderivative of e^x is e^x and so by the constant integration law, an antiderivative of $5e^x$ is $5e^x$.

5. Find an antiderivative of $x + \sqrt{x}$.

> **Solution:** An antiderivative of x is $\frac{x^2}{2} + 5$ and an antiderivative of $\sqrt{x} = x^{1/2}$ is $\frac{2}{3}x^{3/2} + 10$ and so by the addition integration law, an antiderivative of $x + \sqrt{x}$ is $\frac{x^2}{2} + \frac{2}{3}x^{3/2} + 15$.

6. Find an antiderivative to $8t^3 + 15t^2$.

> **Solution:** An antiderivative of t^3 is $\frac{t^4}{4}$ and an antiderivative of t^2 is $\frac{t^3}{3} + \pi$. So using the constant and addition antiderivative law, we get that an antiderivative of $8t^3 + 15t^2$ is $8\frac{t^4}{4} + 15\frac{t^3}{3} + 15\pi = 2t^4 + 5t^3 + 15\pi$.

7. Find an antiderivative to e.

> **Solution:** We can write $e = e \cdot x^0$. An antiderivative to x^0 is $x+1$ so by the constant antiderivative law, an antiderivative to e is $e(x+1) = ex + e$.

8. Find an antiderivative to $\cos u$.

> **Solution:** One choice is $\sin u + 5$.

9. Find an antiderivative to $\sin(2t)$.

> **Solution:** We want to guess $-\cos(2t)$ but the derivative of $-\cos(2t)$ is $\sin(2t) \cdot 2$ by the chain rule. So we can multiply by one half to get a function that works. So one choice is $\frac{-\cos(2t)}{2}$.

Riemann Sums

Example

10. Using 5 right endpoints, estimate the area under $\frac{1}{x}$ on the interval $[1, 6]$.

> **Solution:** Each rectangle will have a length of $\frac{6-1}{5} = 1$ and so the total area is $\frac{1}{2} \cdot 1 + \frac{1}{3} \cdot 1 + \cdots + \frac{1}{6} \cdot 1 = \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{6}$.

11. **TRUE** False The first derivative can tell you if a right endpoint Riemann sum is an overestimate or underestimate.

12. True **FALSE** Left and right endpoint are the only kind of Riemann sums.

Problems

13. Using 5 left endpoints, estimate the area under $\frac{1}{x}$ on the interval $[1, 6]$.

Solution: Each rectangle will have a length of $\frac{6-1}{5} = 1$ and so the total area is $\frac{1}{1} \cdot 1 + \frac{1}{2} \cdot 1 + \cdots + \frac{1}{5} \cdot 1 = \frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{5}$.

14. Using 6 left endpoints, estimate the area under $\frac{1}{x}$ on the interval $[1, 4]$.

Solution: Each rectangle will have a length of $\frac{4-1}{6} = \frac{1}{2}$ and so the total area is $\frac{1}{1} \cdot \frac{1}{2} + \frac{1}{3/2} \cdot \frac{1}{2} + \cdots + \frac{1}{7/2} \cdot \frac{1}{2} = \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{7}$.

15. Using 6 right endpoints, estimate the area under $\frac{1}{x}$ on the interval $[1, 4]$.

Solution: Each rectangle will have a length of $\frac{4-1}{6} = \frac{1}{2}$ and so the total area is $\frac{1}{3/2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} + \cdots + \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{8}$.